COMPUTATIONAL MODEL OF TURBULENT GAS-DISPERSE JET FLOWS

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Using closure of gradient type, a model is constructed for describing the characteristics of the host flow and of the disperse phase. The model is valid in a wide range of variation of particle sizes.

The available methods of modeling turbulent disperse (two-phase) flows can be divided into two groups. The first consists of studies based on the mixed Euler—Lagrange description of motion of the medium: the equations of motion of the continuous host phase are represented and solved in Eulerean variables, and the equations of motion of the disperse phase – in Lagrangian variables, i.e., they are integrated along separate particle (drop) trajectories. Account of the stochastic nature of particle motion within this approach [1-4] leads to a substantial increase in the volume of computation, since for obtaining statistically reliable information it is necessary to have a sufficient representative ensemble of realizations. With decreasing particle size the number of realizations required for obtaining statistically reliable averaging characteristics must, generally speaking, increase, since the contribution of particle interactions with eddies increases with smaller sizes. Therefore, the application of statistical modeling of the dynamics of separate particles is, apparently, advisable only for relatively inert particles ($\tau/T \gtrsim 1$).

The second group covers studies using the Euler representation of the equations of motion for both phases. This direction has been developed intensely in recent years, since it has a number of substantial advantages in comparison with the Euler—Lagrange approach: firstly, a single computational algorithm is used to solve the whole system of equations; secondly, the numerical implementation of determining the turbulent (fluctuating) characteristics of not only the host, but also the disperse phase, is relatively simple. Besides, describing the dynamics of very small particles does not cause any principal difficulties, since the limiting transition to the problem of turbulent diffusion of noninertial impurities is realized for $\tau/T \to 0$.

Most successes in modeling turbulent jet flows of gas suspensions within the Eulerean approach have been achieved in the studies [3-7]. One of the major problems in this case is the determination of turbulent stresses in the disperse phase. In [5, 6] the correlation moments of velocity fluctuations of the disperse phase are directly expressed in terms of Reynolds stresses of the host flow. These expressions have been obtained within the local-homogeneous approximation, and are therefore valid for relatively small particles ($\tau/T \lesssim 1$) in the absence of large velocity gradients of the solid phase in the flow. Relations of gradient type, similar to the Boussinesq hypothesis in a single-phase turbulent flow, are used in the studies [3, 4, 7-10] to determine turbulent stresses in the disperse phase. It must be noted that models of this type are constructed purely phenomenologically on the basis of the analogy with the corresponding transport characteristics in single-phase flow, and therefore, as a rule, contain a large number of empirical constants.

A simple model of the gradient type, constructed by the use of equations for the second moments of velocity fluctuations of the disperse phase, is suggested in the present study for calculating turbulent jet flow of gas suspensions within the Euler approach. Flows are considered with a low bulk concentration of a solid impurity, when particle collisions can be neglected and, consequently, stochastic particle motion is due only to their involvement in the fluctuating motion of the host flow. Particle motion is not accounted for, imposing

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certain restrictions on the validity region of the suggested model. The model constructed does not contain additional empirical constants related to the presence of a disperse phase. For relatively small particles it transforms to the diffusion–migration model [11].

1. The mass and momentum conservation equations of the host and disperse phases under conditions of constant physical properties and a low bulk particle concentration ($\phi \ll 1$) are written in the following form within mechanics of mutually penetrable media

$$\frac{\partial u_k}{\partial x_k} = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k} - \frac{\rho_p \varphi}{\rho \tau}(u_i - v_i) + F_i, \tag{2}$$

$$\frac{\partial \varphi}{\partial t} + \frac{\partial \varphi v_k}{\partial x_k} = 0, \tag{3}$$

$$\frac{\partial \varphi v_i}{\partial t} + \frac{\partial \varphi v_i v_k}{\partial x_x} = \varphi\left(\frac{u_i - v_i}{\tau} + F_i\right). \tag{4}$$

Since the density of particle material is substantially higher than the gas density ($\rho_p \gg \rho$), Eqs. (2) and (4) include only the interphase interaction force within the standard approximation. The dynamic particle relaxation time is determined by the relation

$$\tau = \frac{\rho_p d_p^2}{18\rho\nu(1 + 0.15\,\mathrm{Re}_p^{0.687})} \quad \text{for} \quad \mathrm{Re}_p \leqslant 10^3.$$

We average Eqs. (1)-(4) over an ensemble of turbulent realizations. In that case it is required that the terms containing the bulk impurity concentration satisfy the condition $\langle \phi v_i' \rangle = 0$, i.e., ϕ is used as a weight function, similarly to the well-known Favre averaging method in the theory of single-phase flows with varying density. Thus, the averaged mass and momentum conservation equations of a gas-disperse flow acquire the form

$$\frac{\partial U_k}{\partial x_k} = 0, \tag{5}$$

$$\frac{\partial U_i}{\partial t} + U_k \frac{\partial U_i}{\partial x_k} = -\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_k \partial x_k} - \frac{\partial}{\partial x_k}\langle u_i' u_k' \rangle -$$

$$- \frac{\rho_p}{\rho\tau}[\Phi(U_i - V_i) + \langle \varphi' u_i' \rangle] + F_i, \tag{6}$$

$$\frac{\partial \Phi}{\partial t} + \frac{\partial \Phi V_k}{\partial x_k} = 0, \tag{7}$$

$$\frac{\partial \Phi V_i}{\partial t} + \frac{\partial \Phi V_i V_k}{\partial x_k} = -\frac{\partial \Phi \langle v_i' v_k' \rangle}{\partial x_k} + \Phi\left(\frac{U_i - V_i}{\tau} + F_i\right) + \frac{\langle \varphi' u_i' \rangle}{\tau}. \tag{8}$$

The turbulent stresses in the gas phase are defined by the relation

$$\langle u_i' u_k' \rangle = \frac{2}{3}\delta_{ik} k - \nu_t\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right). \tag{9}$$

The coefficient of turbulent gas viscosity is calculated on the basis of the two-parameter k–ε model of turbulence, according to which

$$\nu_t = C_\mu k^2/\varepsilon. \tag{10}$$

For the correlation $\langle \phi' u_i' \rangle$ appearing in (6), (8) we have used the following gradient representation [11]:

$$\langle \varphi' u_i' \rangle = -\tau g \langle u_i' u_k' \rangle \frac{\partial \Phi}{\partial x_k}, \quad g = \frac{1}{\tau}\int_0^\infty \left[1 - \exp\left(-\frac{s}{r}\right)\right] F(s)\,ds. \tag{11}$$

Taking into account relationship (10), Eq. (8) acquires the form

$$\frac{\partial \Phi V_i}{\partial t} + \frac{\partial \Phi V_i V_k}{\partial x_k} = -\frac{\partial \Phi \langle v_i' v_k' \rangle}{\partial x_k} + \Phi\left(\frac{U_i - V_i}{\tau} + F_i\right) - g\langle u_i' u_k' \rangle \frac{\partial \Phi}{\partial x_k}. \tag{12}$$

The last term in Eq. (12) has a large value for small particles only, since $g \to T/\tau$ for $\tau/T \to 0$. For large particles the contribution of this term becomes unimportant, since $g \sim (T/\tau)^2$ for $\tau/T \to \infty$. To simplify the description of the original system of equations we represent the velocity of motion of the disperse phase in the form

$$V_i = V_{0i} - \tau g \langle u_i' u_k' \rangle \frac{\partial \ln \Phi}{\partial x_k}, \tag{13}$$

where V_{0i} satisfies the equation

$$\frac{\partial \Phi V_{0i}}{\partial t} + \frac{\partial \Phi V_{0i} V_{0k}}{\partial x_k} = -\frac{\partial \Phi \langle v_i' v_k' \rangle}{\partial x_k} + \Phi \left(\frac{U_i - V_{0i}}{\tau} + F_i \right). \tag{14}$$

The second term on the right hand side of Eq. (13) describes the "diffusion" velocity of small particles and its contribution to the total velocity V_i decreases with increasing particle inertia. We note that, unlike V_i, the particle velocities V_{0i} tend to the gas velocity with their decreasing inertia, i.e., $V_{0i} \to U_i$ when $\tau \to 0$. With account of (13), Eq. (7) for the disperse phase concentration is written in the form of a diffusion equation

$$\frac{\partial \Phi}{\partial t} + \frac{\partial \Phi V_{0k}}{\partial x_k} = \frac{\partial}{\partial x_i} \left[\left(\frac{T_p}{T} - \frac{\tau}{T} f \right) D_{ik} \frac{\partial \Phi}{\partial x_k} \right]. \tag{15}$$

With account of (11) and (13), the term due to the reciprocal particle effect on the average motion of the host flow in Eq. (6) is

$$A_{U_i} = \frac{\rho_p}{\rho \tau} [\Phi (U_i - V_i) + \langle \varphi' u_i' \rangle] = \frac{\rho_p \Phi}{\rho \tau} (U_i - V_{0i}). \tag{16}$$

To determine the turbulent stresses in the disperse phase $\langle v_i' v_k' \rangle$ we use the transport equations of second moments, obtained from (3), (4):

$$\frac{\partial \langle v_i' v_k' \rangle}{\partial t} + V_n \frac{\partial \langle v_i' v_k' \rangle}{\partial x_n} + \langle v_i' v_n' \rangle \frac{\partial V_k}{\partial x_n} + \langle v_k' v_n' \rangle \frac{\partial V_i}{\partial x_n} + \frac{1}{\Phi} \frac{\partial \Phi \langle v_i' v_k' v_n' \rangle}{\partial x_n} = \frac{\langle v_i' u_k' \rangle + \langle v_k' u_i' \rangle - 2 \langle v_i' v_k' \rangle}{\tau}. \tag{17}$$

The mixed correlation moment of velocity fluctuations of the solid and gas phases is related, within the locally homogeneous approximation, to the Reynolds stresses of the gas by the well-known relation [6, 12]

$$\langle v_i' u_k' \rangle = f \langle u_i' u_k' \rangle. \tag{18}$$

With account of (18), it follows from (17) that

$$\langle v_i' v_k' \rangle = f \langle u_i' u_k' \rangle - \frac{\tau}{2} \left(\langle v_i' v_n' \rangle \frac{\partial V_{0k}}{\partial x_n} + \langle v_k' v_n' \rangle \frac{\partial V_{0i}}{\partial x_n} + \frac{\partial \langle v_i' v_k' \rangle}{\partial t} + V_{0n} \frac{\partial \langle v_i' v_k' \rangle}{\partial x_n} + \frac{1}{\Phi} \frac{\partial \Phi \langle v_i' v_k' v_n' \rangle}{\partial x_n} \right). \tag{19}$$

We replace V_i by V_{0i} in expression (19). This replacement does not lead to a substantial error, since the total contribution to $\langle v_i' v_k' \rangle$ of terms in the circular brackets is proportional to τ, and, consequently, becomes substantial only for relatively large particles, when the difference between V_i and V_{0i} vanishes.

With the purpose of simplifying expression (19), we use isotropic representations for its terms in the right hand side:

$$\frac{\partial \langle v_i' v_k' \rangle}{\partial t} + V_{0n} \frac{\partial \langle v_i' v_k' \rangle}{\partial x_n} + \frac{1}{\Phi} \frac{\partial \Phi \langle v_i' v_k' v_n' \rangle}{\partial x_n} = \frac{\delta_{ik}}{3} \left(\frac{\partial \langle v_j' v_j' \rangle}{\partial t} + V_{0n} \frac{\partial \langle v_j' v_j' \rangle}{\partial x_n} + \frac{1}{\Phi} \frac{\partial \Phi \langle v_j' v_j' v_n' \rangle}{\partial x_n} \right),$$

$$\langle v_i' v_k' \rangle = \frac{\delta_{ik}}{3} \langle v_n' v_n' \rangle.$$

With account of these relations, expression (19) for the turbulent stresses in the disperse phase acquires the form

$$\langle v_i' v_k' \rangle = \frac{2}{3}\delta_{ik}k_p - \nu_p\left(\frac{\partial V_{0i}}{\partial x_k} + \frac{\partial V_{0k}}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial V_{0n}}{\partial x_n}\right), \tag{20}$$

where the turbulent viscosity coefficient of particles equals

$$\nu_p = f\nu_t + \tau k_p/3. \tag{21}$$

The turbulent energies of the solid and gas phases are related within the locally homogeneous approximation by the relation

$$k_p = fk. \tag{22}$$

According to (21) and (22), the following limiting relations are valid for the turbulent viscosity of the disperse phase

$$\nu_p \to \nu_t \quad \text{for} \quad \tau/T_p \to 0 \quad (f \to 1),$$
$$\nu_p \to T_p k/3 \quad \text{for} \quad \tau/T_p \to \infty \quad (f \to T_p/\tau).$$

The equations for the turbulent gas energy and its dissipation in the presence of particles follow from (1), (2). They coincide with the corresponding equations for single-phase flow, except for additional terms due to the interphase interaction. Without accounting for fluctuations in particle concentration, these terms are written in the form

$$A_k = \frac{2\rho_p\Phi(1-f)k}{\rho\tau}, \quad A_\varepsilon = \frac{2\rho_p\Phi(1-f_\varepsilon)\varepsilon}{\rho\tau}. \tag{23}$$

Approximating F(s) and F_ε(s), as in [12, 13], by staggered functions, the following expressions are obtained for the coefficients of particles involved in the macro- and microfluctuating motion of the host flow:

$$f = 1 - \exp(-T_p/\tau), \quad f_\varepsilon = 1 - \exp(-T_\varepsilon/\tau).$$

The interaction time of particles with the energy emitting gas fluctuations is determined by the approximation equation

$$T_p = \frac{T}{\sqrt{1 + (T|\mathbf{U} - \mathbf{V}|/L)^2}},$$

satisfying the limiting relation

$$T_p \to T \quad \text{for} \quad \frac{T|\mathbf{U}-\mathbf{V}|}{L} \to 0, \quad T_p \to \frac{L}{|\mathbf{U}-\mathbf{V}|} \quad \text{for} \quad \frac{T|\mathbf{U}-\mathbf{V}|}{L} \to \infty.$$

The microscale turbulence time under the assumption of isotropy of small-scale fluctuating motion is determined by the relation $T_\varepsilon = (15\nu/\varepsilon)^{1/2}$.

2. We represent the system of equations for calculating stationary axially symmetric flow of gas suspensions within the approximations of boundary layer theory. The equations of motion of the gas and solid phases (5), (6), (14), (15), with account of relations (16) for the interphase interaction, are written in the form

$$\frac{\partial U_x}{\partial x} + \frac{1}{r}\frac{\partial rU_r}{\partial r} = 0, \tag{24}$$

$$U_x\frac{\partial U_x}{\partial x} + U_r\frac{\partial U_x}{\partial r} = -\frac{\partial P}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left[r(\nu+\nu_t)\frac{\partial U_x}{\partial r}\right] - \frac{\rho_p\Phi(U_x - V_{0x})}{\rho\tau} + F_x, \tag{25}$$

$$\frac{\partial \Phi V_{0x}^2}{\partial x} + \frac{1}{r}\frac{\partial r\Phi V_{0r}V_{0x}}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\Phi\nu_p\frac{\partial V_{0x}}{\partial r}\right) + \Phi\left(\frac{U_x - V_{0x}}{\tau} + F_x\right),$$

$$\frac{\partial \Phi V_{0x}V_{0r}}{\partial x} + \frac{1}{r}\frac{\partial r\Phi V_{0r}^2}{\partial r} = -\frac{2}{3}\frac{\partial \Phi k_p}{\partial r} + \Phi\left(\frac{U_r - V_{0r}}{\tau} + F_r\right), \tag{26}$$

(27)

$$\frac{\partial \Phi V_{0x}}{\partial x} + \frac{1}{r}\frac{\partial r\Phi V_{0r}}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{T_p}{T} - \frac{\tau}{T}f\right)D_t\frac{\partial \Phi}{\partial r}\right]. \tag{28}$$

In writing down Eqs. (27), (28) it is taken into account that the structure of fluctuating characteristics of jet flows is nearly isotropic, and the following relations are used accordingly

$$\langle u_r'^2 \rangle = 2k/3, \quad \langle v_r'^2 \rangle = 2k_p/3, \quad D_t = T \langle u_r'^2 \rangle = 2Tk/3.$$

The integral time of gas velocity fluctuation scales is determined by the expression

$$T = \alpha \frac{k}{\varepsilon}, \quad \alpha = \frac{3C_\mu}{2\,\mathrm{Sc}_t}.$$

For relatively small particles, i.e., for decreasing inertial parameter τ/T, the system of equations (26)-(28) transforms to the diffusion-migration model of describing the propagation of slightly disperse impurities in a turbulent flow [11].

With account of the terms of (23) resulting from the reciprocal particle effect on turbulence, fluctuating interphase gliding, the balance equations of turbulent gas energy and its dissipation are written in the form

$$U_x \frac{\partial k}{\partial x} + U_r \frac{\partial k}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial r} \right) + \nu_t \left(\frac{\partial U_x}{\partial r} \right)^2 - \varepsilon - 2 \frac{\rho_p}{\rho\tau} \Phi (1 - f)\, k, \tag{29}$$

$$U_x \frac{\partial \varepsilon}{\partial x} + U_r \frac{\partial \varepsilon}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial r} \right) + $$
$$+ C_1 \frac{\varepsilon}{k} \nu_t \left(\frac{\partial U_x}{\partial r} \right)^2 - C_2 \frac{\varepsilon^2}{k} - 2 \frac{\rho_p}{\rho\tau} \Phi (1 - f_\varepsilon)\, \varepsilon. \tag{30}$$

The constants in (10), (29), (30) have standard values [14]: $C_\mu = 0.09$; $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$; $C_1 = 1.44$; $C_2 = 1.92$. According to experimental data for diffusion of noninertial impurities in axially symmetric jets the turbulent Schmidt number is taken equal to 0.8.

As boundary conditions at the jet axis we use the flow symmetry requirement

$$r = 0 \quad U_r = V_{0r} = \frac{\partial U_x}{\partial r} = \frac{\partial V_{0x}}{\partial r} = \frac{\partial k}{\partial r} = \frac{\partial \varepsilon}{\partial r} = 0,$$

and at the external boundary – the equality of mean characteristics of the host and disperse phases of the flow wake parameters

$$r = \delta_e \quad U_x = U_e, \quad V_{0x} = V_e, \quad \Phi = \Phi_e.$$

To determine the boundary conditions for the fluctuating characteristics of the host flow we used the solution of turbulent transport equations in the homogeneous isotropic approximation [13]. The initial conditions were determined either on the basis of experimental data, or by using the solution at the developed portion of the tube for mean and fluctuating quantities of the host phase and the assumption of homogeneity of characteristics of the disperse phase.

3. To test the model suggested, a comparison was carried out with the experimental data of [15-18].

Figure 1 shows a comparison of calculated and experimental data [17] on damping of axial gas and particle velocities along the jet. The presence of a disperse phase leads to an increased elongation of the jet in comparison with single-phase (sp) flow, in which case, due to the inertia of the disperse phase the damping of longitudinal particle velocity oc-

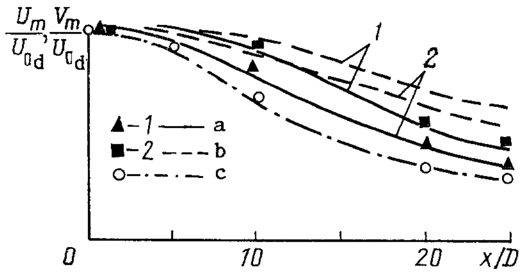


Fig. 1. Damping of longitudinal gas and particle velocities along the jet axis: 1) particle velocity V_m/U_{sp}, 2) gas velocity U_m/U_{sp}, a) $\chi_0 = 0.32$, b) 0.85, c) 0. Experimental data of [17].

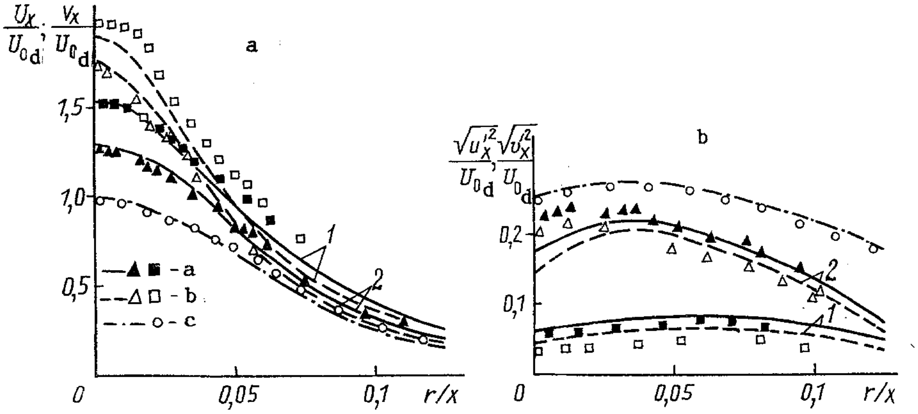
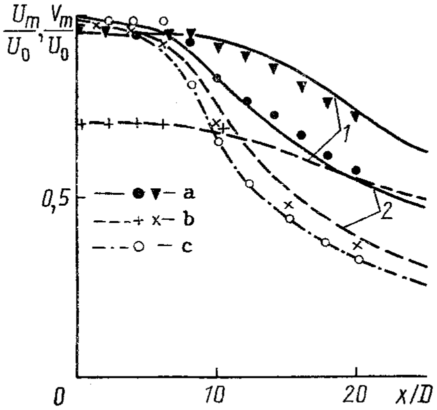


Fig. 2. Radial velocity distributions (a) and fluctuation intensities (b) of the host and disperse phases: 1) particle velocity V_x/U_{sp} (intensity of longitudinal fluctuations of particles $\sqrt{v_x'^2}/U_{0d}$) 2) gas velocity U_x/U_{sp} (intensity of longitudinal fluctuations of gas $\sqrt{u_x'^2}/U_{0d}$) a) $\chi_0 = 0.32$, b) 0.85, c) 0. Experimental data of [17].



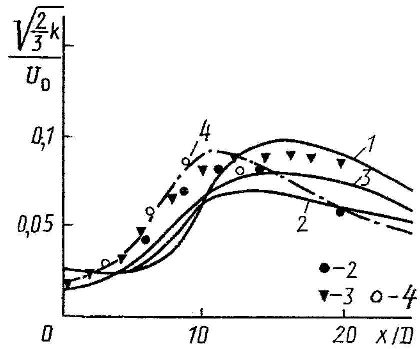


Fig. 3 Fig. 4

Fig. 3. Damping of longitudinal gas and particle velocities along the jet axis: 1) particle velocity V_m/U_0, 2) gas velocity U_m/U_0, a) $d_p = 170\ \mu$, $\chi_0 = 0.86$; b) $d_p = 500\ \mu$, $\chi_0 = 1.85$; a) $\chi_0 = 0$. Experimental data of [18].

Fig. 4. Variation of velocity fluctuation intensity of the host phase over the length of the jet: 1) $d_p = 7\ \mu$, $\chi_0 = 0.22$; 2) $d_p = 17\ \mu$, $\chi_0 = 0.22$, experiment [15], 3) $d_p = 170\ \mu$, $\chi_0 = 0.86$, experiment [18], 4) single-phase jet, experiment [20].

curs more slowly than that of the gas velocity. With increasing χ_0 we have an increase in the jet elongation, which is determined by the laminarization and increase in the total flow momentum [19].

Results of calculating radial distributions of mean velocities of the gas and disperse phase are presented in Fig. 2a. An important effect, obtained both in the calculation and experimentally is the decrease in the jet width, accompanied by an increase in its elongation.

In Fig. 2b we show results of calculating longitudinal fluctuations of the host and disperse phases. One observes substantial action of the disperse phase on the fluctuating flow structure. With increasing particle mass concentration one observes suppression of fluctuations of both the host and the disperse phases, corresponding to study results of other authors [3-7, 15-19].

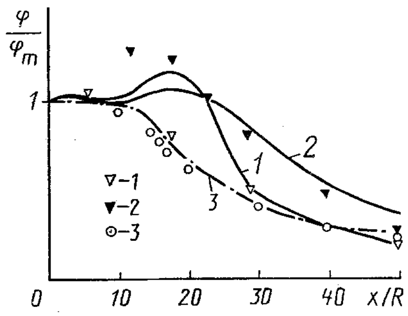


Fig. 5. Variation of concentration of slightly disperse impurities along the jet axis: 1) $d_p = 7$ μm, experiment [15]; 2) $d_p = 17$ μm, experiment [16]; 3) single-phase jet, experiment [21].

The results provided verify the validity of using expression (20) for calculating the stress tensor in the disperse phase and expression (22) for the turbulent energy of particles.

With increasing particle sizes the basic effect of the disperse phase is manifested in the interaction of the mean fields. As shown in Fig. 3, the presence of very large particles in the flow ($d_p = 500$ μm) does not exert substantial influence on the parameters of the host flow, and the damping of the longitudinal gas velocity of two-phase and single-phase jets occur practically identically. This indicates the conservation property of the fluctuating structure of the gas phase with respect to presence of large disperse impurities in the flow (Fig. 4, curves 3 and 4). However, with increasing distance from the jet intensification takes place of the turbulent flow energy, due to inertial transport of host phase fluctuations by particles – as verified by experimental data.

Quite many publications (e.g., [4-7]) are devoted to calculating the dynamics of small particles ($\tau/T < 1$). The basic action mechanism of the host and disperse phases is in this case the mismatch between the gas and particle velocity fluctuations in the absence of mean gliding. Accordingly, the most interesting information on mean characteristics is obtained in calculating concentration fields of the disperse phase. As shown in experimental studies [15, 16], the concentration distribution of inertial impurities along the jet axis has an anomalous nature, manifested in "lacing" effects. In a number of studies [5, 6, 15, 16, 19] this fact is explained by the effect of the Magnus force, acting on a rotating particle. Particle rotation results from emergence conditions of jet flow from a nozzle. As shown in [11], the "lacing" effect, along with rotation, is determined by particle involvement in an inhomogeneous fluctuating flow field. The computational model suggested in the present study is a long-range development of the diffusion–migration model described in [11], and transforms into it when $\tau/T_p \to 0$. Figure 5 shows the variation of concentration of small particle impurities over the length of the jet. The "lacing" effect obtained in the calculations is not expressed as explicitly as in the experimental studies. In our opinion, this effect is determined by the high flow velocities of the jet ($U_0 \approx 45$ m/sec), for which one can have substantial effects of particle rotation in the jet, generated by collisions with the wall.

Due to the different action of particles on turbulent eddies of inertial and dissipative scales, the presence of slightly disperse impurities can lead not only to suppression, but also to generation of energy fluctuations of the host phase [13] (see Fig. 4). Unlike large particles, the growth of energy fluctuations is determined in this case by the decrease in additional dissipation in the equations for the dissipation rate of gas flow.

Based on the results provided above, it can be concluded that the model suggested describes adequately the mean and fluctuating characteristics of the host flow and the disperse phase, and is applicable to a wide range of variation of particle sizes and mass concentrations of inertial impurities.

NOTATION

Here t denotes time; x_i are Cartesian coordinates; x, r are axial and radial coordinates; u_i, U_i, u_i' are the actual, mean, and fluctuating gas velocity; v_i, V_i, v_i' are the actual, mean, and fluctuating particle velocities; τ is the time of dynamic particle relaxation; ϕ, Φ, are the actual and mean bulk particle concentrations; p, P are the actual and mean pressures; ρ, ρ_p, are the gas and particle material densities; ν is the kinematic viscosity coefficient of the gas; F_i is the acceleration of the mass force (e.g., gravity force); d_p is the particle diameter; T is the temporal integral scale of turbulence; T_p is the interaction time of particles with turbulent gas fluctuations; T_ε is the temporal microscale of turbulence; $D_{ik} = T\langle u_i' u_k' \rangle$ is the turbulent diffusion tensor of noninertial impurities, D_t is the diffusion coefficient of noninertial impurities; $Sc_t = \nu_t/D_t$ is the turbulent Schmidt number; $L = (2k/3)^{1/2}T$ is the spatial integral scale of turbulence; $k = \langle u_n' u_n' \rangle/2$ is the turbulent gas energy; $\varepsilon = \nu\langle(\partial u_i'/\partial x_k)^2\rangle$ is the dissipation of turbulent gas energy; $Re_p = d_p|U - V|/\nu$, is the Reynolds number of interphase gliding; $\langle v_i' v_k' \rangle = \langle \varphi v_i' v_k' \rangle/\Phi$ are turbulent stresses in the disperse phase; $k_p = \langle v_n' v_n' \rangle/2$ is the turbulent energy of particles $f = 1/\tau \int_0^\infty \exp(-s/\tau) F(s)\, ds$; $f_\varepsilon = 1/\tau \int_0^\infty \exp(-s/\tau) F_\varepsilon(s)\, ds$ are the particle involvement coefficients in the macro- and microfluctuating motion; $g = T/\tau - f$; $F(s) = \langle u_i'(t) u_k'(t+s)\rangle/\langle u_i'(t) u_k'(t)\rangle$ the two-time correlation function of gas fluctuation velocity along the particle trajectory; $F_\varepsilon(s) = \langle \partial u_i'(t)/\partial x_k \cdot \partial u_i'(t+s)/\partial x_k \rangle/\langle \partial u_i'(t)/\partial x_k \cdot \partial u_i'(t)\, \partial x_k \rangle$ is the correlation function of the derivatives of gas velocity fluctuations; D is the jet diameter; δ_e is the coordinate of the jet boundary; and $\chi_0 = \Phi_0 \rho_p/\rho$ denotes the mass concentration of the disperse phase. Subscripts: 0, value at the jet cross section; e, value at external flow; sp, characteristics of single-phase flow; m, value at jet axis.

LITERATURE CITED

1. D. L. Bulzan, J. S. Suen, and G. M. Faeth, AIAA Paper #303, 1-26 (1987).
2. P. Desjonqueres, A. Berlemont, and G. Gousbet, J. Aerosol. Sci., 19, No. 1, 99-103 (1988).
3. A. Adeniji-Fasholoa and C. P. Chen, AIAA Paper #1347, 1-8 (1987).
4. A. A. Mostafa and H. C. Mongia, Int. J. Heat Mass Trans., 30, No. 12, 2583-2593 (1987).
5. L. B. Gavin and V. A. Naumov, Dokl. Akad. Nauk SSSR, 283, No. 2, 336-339 (1985).
6. A. A. Shraiber, L. B. Gavin, V. A. Naumov, and V. P. Yatsenko, Turbulent Flow of Gas Suspensions [in Russian], Kiev (1987).
7. C. P. Chen and P. E. Wood, AIChE J., 32, No. 1, 163-166 (1986).
8. F. Pourahmadi and J. A. C. Humphrey, Phys. Chem. Hydrodyn., 4, No. 3, 191-219 (1983).
9. B. Dobrowolski, Arch. Mech., 38, No. 5-6, 611-634 (1986).
10. M. A. Rizk and S. E. Elgobahsi, Int. J. Multiphase Flow, 15, No. 1, 119-133 (1989).
11. A. A. Vinberg, L. I. Zaichik, and V. A. Pershukov, Inzh.-fiz. Zh., 59, No. 4, 609-614 (1990).
12. I. V. Derevich, V. M. Eroshenko, and L. I. Zaichik, Inzh.-fiz. Zh., 45, No. 4, 454-460 (1983).
13. L. I. Zaichik and V. A. Pershukov, Inzh.-fiz. Zh., 58, No. 4, 556-560 (1990).
14. B. E. Launder and D. B. Spalding, Comp. Meth. Appl. Mech. Eng., 3, No. 1, 269-289 (1974).
15. M. K. Laats and F. A. Frishman, Mekh. Zhidk. Gaza, No. 2, 186-191 (1970).
16. M. K. Laats and F. A. Frishman, Mekh. Zhidk. Gaza, No. 2, 153-157 (1973).
17. D. Modarress, Kh. Tan', and S. El'khobashi, AKT, 2, No. 12, 83-91 (1984).
18. V. Tsuji, Y. Morikawa, T. Tanaka, et al., Int. J. Multiphase Flow, 14, No. 5, 565-574 (1988).
19. G. N. Abramovich (ed.), Theory of Turbulent Jets [in Russian], Moscow (1984).
20. L. Boguslawsky and Cz. O. Popiel, J. Fluid Mech., 90, No. 3, 531-539 (1979).
21. V. A. Golubev and V. F. Klimkin, Inzh.-fiz. Zh., 34, No. 3, 493-499 (1978).